Comparison of the rate of cosmic ray and collider experiments

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The intensity of cosmic rays depending on energy is often displayed in diagrams using the differential form. For non-specialists it is difficult to extract total numbers for collisions. There are also no diagrams that compare the terestrial production rate of high energy collisions for nature and experiments in a single picture. In this paper I will focuse only on natural collisions due to cosmic rays that occur in the upper atmosphere of the earth. The resulting diagram is displayed at the end of this paper.

1 Cosmic rays

The intensity of the cosmic ray is mostly given in a differential form.

$$I_N(E)\left[\frac{particles}{m^2\,s\,sr\,GeV}\right]$$

To calculate the number of particles an integration over area, time, solid angle and energy has to be done.

1.1 Integration over energy

The integration over energy can be simplified because the intensity follows a potence law

$$I_N(E)\approx I_N(E_0)\left(\frac{E}{E_0}\right)^{-\alpha} \quad .$$

The Integration over Energy from Energie E_0 to E_1 is then given by

$$\int_{E_0}^{E_1} I_N(E)\,dE \approx I_N(E_0)\left[\frac{E_0}{(1-\alpha)}\left(\frac{E_1}{E_0}\right)^{1-\alpha} - \frac{E_0}{(1-\alpha)}\left(\frac{E_0}{E_0}\right)^{1-\alpha}\right] \quad .$$

The term $(E_1 / E_0)^{1-\alpha}$ vanishes fast for $\alpha \approx 2{,}7$, so the integral can be replaced by an integral from energy E_0 to infinity

$$\int_{E_0}^{\infty} I_N(E)\,dE \approx I_N(E_0)\frac{E_0}{(\alpha-1)} \quad .$$

It is common to use always the number of collisions from a certain energy till infinity. As the rate drops fast with rising energy, this number is not much higher than the value for reasonable smaller intertervalls.

1.2 Integration over space angle

To calculate the number of particles that pass a certain Area A from one side an integration over the solid angle has to be done. Assuming that the intensity is isotrop the flux through a surface by particles from one side F_N is given by

$$F_N = \pi I_N \quad .$$

This can be derived in a simple way for a sphere. The Surface S of a sphere with radius R is given by

$$S = 4\pi R^2 \quad .$$

Rays comiming from one direction will hit the sphere if they pass through a circle of area A

$$A = \pi R^2 \quad .$$

The number of particles that hit this sphere is given by

$$\frac{dN}{dt} = I\, A d\Omega = I\, \pi\, R^2\, d\Omega \quad .$$

The solid angle of a full sphere is 4π. So replacing dΩ with 4π gives the number of particles that hit the sphere

$$\frac{dN}{dt} = I\, \pi\, R^2\, 4\pi \quad .$$

To derive the number of particles that hit a unit square of the surface, the rate is divided by the surface of the sphere S = 4 π R^2

$$\frac{dN}{dt}=\frac{I\pi R^2 4\pi}{4\pi R^2}=\pi I \quad .$$

1.3 Integrated diagram for cosmic rays

Using the above formulas, the well known diagram for the flux of cosmic rays over energy, can be displayed in a different form. The data for the flux of cosmic rays is taken from [Nakamura 2010] Chapter *24. Cosmic Rays* Figures 24.1 and 24.8.

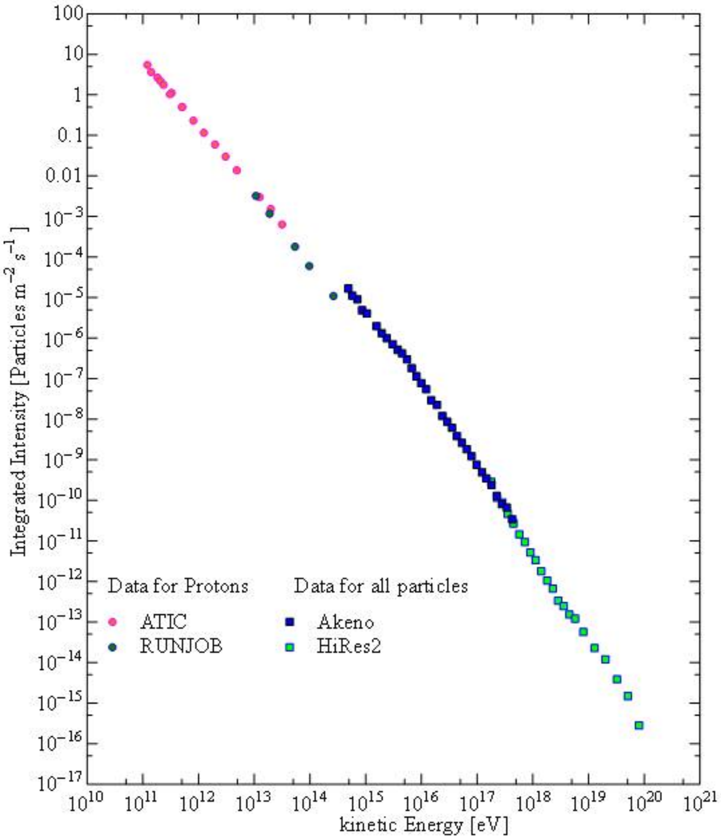

Figure 1: The integrated flux oc cosmic rays. The integration is done using a constant differential spectral index α = 2.7. For lower energy data for individual components are available. Data from the balloon experiments ATIC and RUNJOB for protons are given. For higher energies only the total flux is known and the data is for the energy which should be approximately equal of the kinetic energy. Given is the total flux of cosmic rays that hit a given surface from one side.

Figure 1 by itself is helpful. It is easy to get the absolute number of collisions for a certain energy. Other than in the convential diagrams, no integration over energy has to be done. The drawback is that assumptions about the spectral index have to be made.

2 Collision rate in collider experiments

In colliders rays of particles of equal energy are directed against each other. The collision rate is then dependend from the luminosity L by multiply with the cross section σ

$$\frac{dN}{dt} = \sigma L \quad .$$

The achieved Luminosity in the experiments is not constant. To calcluate the number of collisions it is therefore practical to use the integrated Luminosity

$$N = \sigma \int L \, dt \quad .$$

Often the unit barn (b) is used. It is

$$1\, b = 10^{-24} cm^2 \quad .$$

To calculate the rate of collisions per year at a given energy the operation time t_{op} has to be known.

Collider Name (Year of operation)	**Energy per ray (GeV)**	**Peak Luminosity ($cm^{-2}\, s^{-1}$)**	**Integrated Luminosity (fb^{-1})**	**Collisions**	**Assumed Working time with high luminosity**	**Collisions per year**
Tevatron Run II	980	$2 \cdot 10^{32}$	12	$1.2 \cdot 10^{15}$	6	$2 \cdot 10^{14}$
LHC (2010)	3500	$5 \cdot 10^{31}$	0.047	$4.7 \cdot 10^{12}$	1	$4.7 \cdot 10^{12}$
LHC (2011)	3500	10^{32}	12.57	$1.3 \cdot 10^{15}$	1	$1.3 \cdot 10^{15}$
LHC (projected)	7000	10^{34}	1000	10^{17}	10	10^{16}

Sources:

Tevatron: [CDF] and [Luminosity Run II].

LHC (2010) Luminosity - delivered 47.03 pb-1

recorded 43.17 pb-1 [Lamont 2010]

LHC (2011) Luminosity [lpc 2011]

For the LHC in 2011 the expected integrated luminosity ∫Ldt is expected to be 10 fb^{-1}. with σ = 100mb the yearly collision rate is given by N = σ ∫Ldt, so $N = 10^{15}$.

LHC General: [Lamont 2010b]

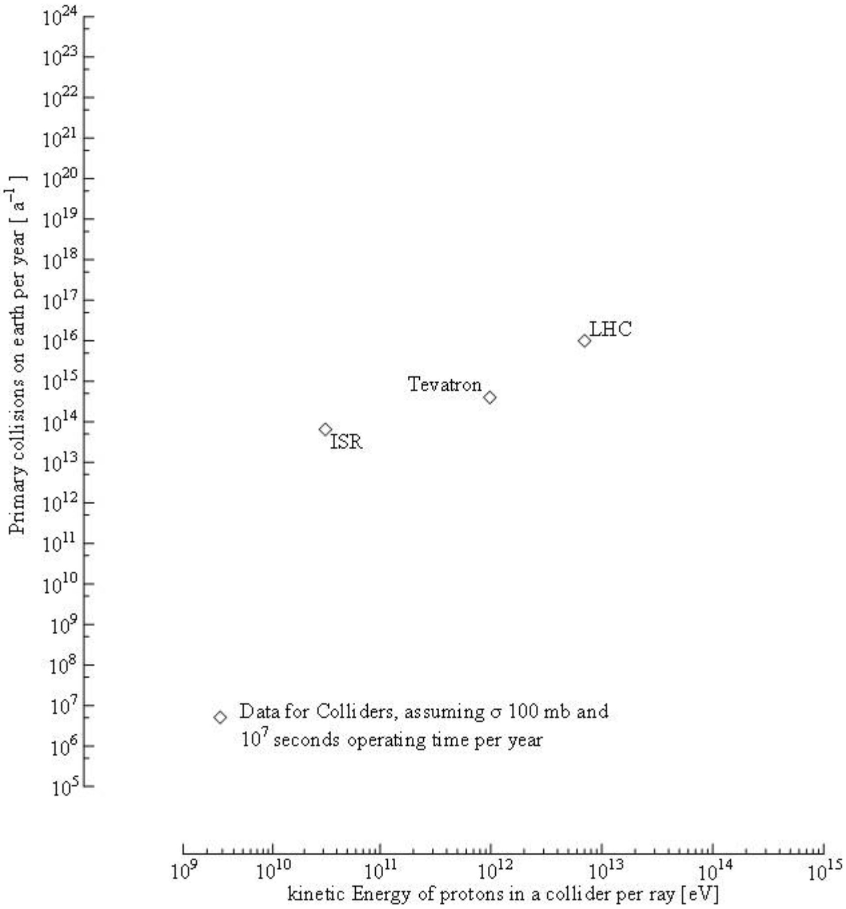

Figure 2: Average number of collisions in collider experiments per year during their active operating time. The operating time is assumed to be 10 million seconds, the cross section σ=100mb. The energy axis is nonlinear and scaled in such a way, that a direct comparison with cosmic rays that hit a atom at rest can be done. Data for ISR are tentative and not referenced in this document.

3 Comparison of experimental and natural collisions on earth

The above shown diagrams can now be combined to a new single diagram. For colliders the energy per ray is given. This rays collide frontally, the total energy in the rest system is therefore twice this value. For cosmic ray the total energy in the rest system can be calculated using the Special Theory of Relativity. This is not yet explained in detail in this paper.

In the next diagram both pictures are combined.

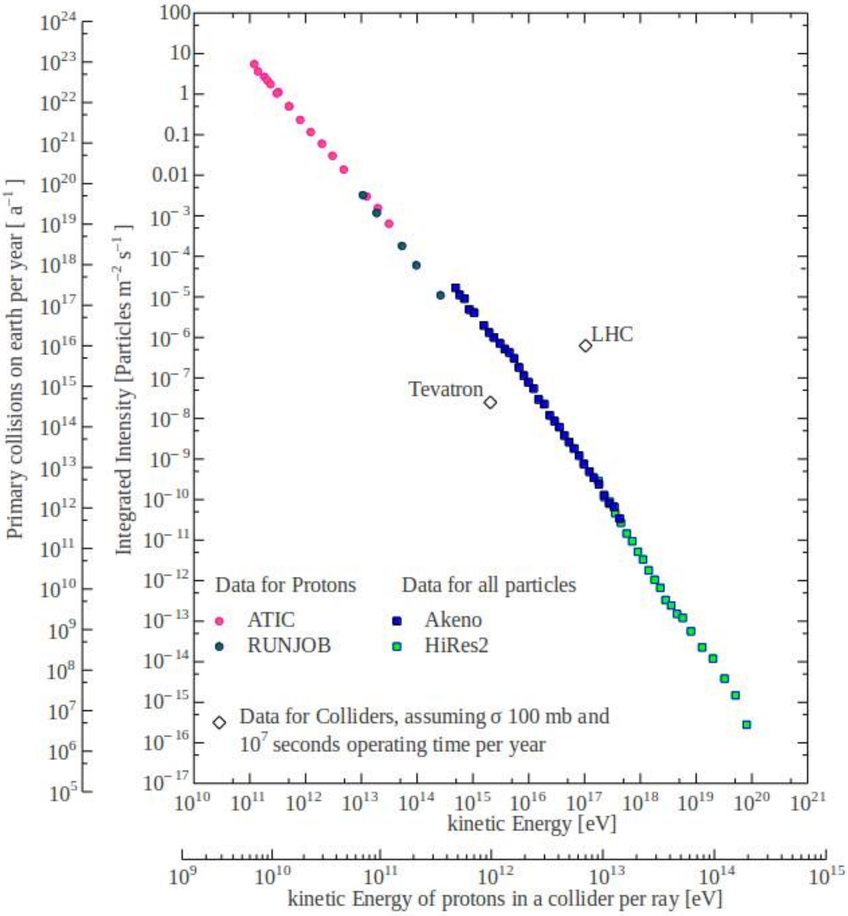

Abbildung 1: Figure 3 Comparison of the terrestrial rate of collisions due to cosmic rays and experimental ones. For ISR, Tevatron and LHC the average number of collisions in collider experiments per year during their active operating time is given. The operating time is assumed to be 10 million seconds, the cross section σ=100mbShown is also the integrated flux of cosmic rays. The integration is done using a constant differential spectral index α = 2.7. For lower energy, data for individual components are available. Data from the balloon experiments ATIC and RUNJOB for protons are given. For higher energies only the total flux is known and the data is for the energy which should be approximately equal of the kinetic energy.For ISR and Tevatron the points lay under the line for cosmic rays. The majority of collisions with the corresponding center of mass energy next to earth is therefore natural. For LHC at the planned luminosity and energy the points is well above the line. The majority of collisions during the runtime next to earth will therefore occur in the experiment.

References

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